Mathematics

At

Heamoor School

January 2024

Intent

At Heamoor School we aim to provide a maths curriculum that is engaging and creative; one that inspires confidence in every child so that they become resilient and independent learners. We aim to develop the skills necessary for children to use and apply their knowledge both in lessons and in the outside world. They will understand problems, become 'deep thinkers' and be able to reason about the world around them. They will know the purpose of their learning, relate it to real life and use it in their everyday lives. Most importantly they will develop a love of maths that is evident in the 'buzz' for learning that exists for each child.

Implementation

At Heamoor School we use a range of resources (for example White Rose Planning, NCETM, NRICH, Testbase) to give our children full coverage of the Maths National Curriculum ensuring that topics are revisited as needed and that knowledge is fully embedded. High quality teaching, following the Mastery approach, delivers appropriately planned lessons for all children offering support and challenge for individuals. Lessons support fluency, reasoning and problem solving which give children opportunities to practise, consolidate and explore ideas. Maths lessons are delivered in creative and engaging ways using a wide variety of resources to support key concepts so that children are able to form the images they need to develop their understanding. Children are encouraged to use these manipulatives whenever they are

needed no matter how old they are. Lessons are designed to challenge and inspire the children and the importance of using maths in all areas of the curriculum is fully recognised by staff. Children are encouraged to talk, reason, explain and justify their answers whenever possible in order to develop their thinking and confidence in maths, and beyond. Working walls support the use of correct vocabulary and effective methods and the children use these daily. The structure of the maths lesson is flexible but will usually include: key vocabulary; skills practise; some modelling by the teacher; independent work and opportunities to reason; challenge. A topic may start with a 'hook' which provides some context for the learning and engages the children right from the start. At Heamoor School our aim, in all aspects of school life, is to inspire, engage and challenge children to try new things, have a go without fear of mistakes and persevere with tricky tasks – our implementation of maths reflects this.

Impact

We measure the impact of our maths curriculum in terms of achievement and progress. We use summative assessments (White Rose end of unit tests & end of term assessments) and formative judgements (teacher assessment using Insight) to make informed decisions about the children and decide on their next steps. For all of us at Heamoor School though, the impact of the maths curriculum is about more than this. It is about the confidence and attitude of the children, their engagement in learning and their enthusiasm for the subject. It is these attributes that we truly aim to foster in the children (and the adults), not just in maths but in every aspect of life.

Times Tables

At Heamoor School, the times tables scheme is intended to be repetitive and cyclical so that children are constantly practising their fluency and understanding of times tables whilst at the same time building their tables knowledge. Each stage builds on the one before and adds on additional tables.

Children will be encouraged to practise tables with their parents at home and work towards a Bronze, Silver or Gold certificate in each of the four stages.

Children will receive a Bronze certificate when they can say all the tables, in their stage, in order – the teacher will initial the first blue box of their sheet for each table to show this has been achieved.

A Silver certificate will be awarded when the children can answer tables questions given in any order both verbally and in written form, e.g. 2x8, 2x2 etc. The teacher will initial the 2nd blue box for every table when this has been achieved.

A Gold certificate will be awarded when the children can show that they know related division facts for every table in their stage both verbally and in written form, e.g. 18 divided by 2, or when given an answer they can tell you the question e.g. when given 20 they know it is 2 x 10.

When the children have completed a stage, they can then move on to the next stage of the tables scheme.

Presentation in Books

In Key Stage 1, children are expected to:

- Write one digit in a square.
- Use a ruler to draw straight lines.

In Key Stage 2, children are expected to:

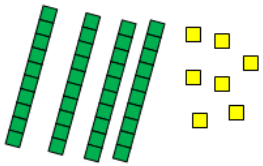
- Write the short date and underline it, at the start of each piece of work.
- Write the Learning Intention clearly and underline it (KS1 may use stickers to support with this).
- Draw a margin 2/3 squares in from the left-hand side using a ruler.
- Write one digit in a square.
- Use a ruler to draw straight lines.

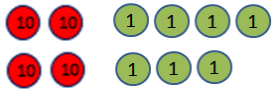
Mental Calculation and Question Types

Key to every child's mathematical confidence and success is the ability to visualise problems and their solutions. When working mentally there are many strategies that may support the children when solving a variety of questions and they should be encouraged to have a go, make mistakes and correct their thinking through the use of the following:

- **<u>Manipulatives</u>** can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related eg.

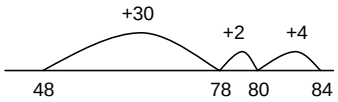
  What's the same? What's different?





- **<u>Blank Number Lines</u>**

e.g. 48 + 36 =



- **<u>Informal Jottings</u>**

Children are encouraged to use informal jottings to help them make sense of their thoughts and keep track of the different steps involved in solving problems. These are separate from formal written methods (see Presentation in Books) and should allow the children to develop ideas in a manner that suits them - it may be pictorial, it may be numbers etc. Children should be able to follow their jottings and explain their thoughts but this is not 'marked' in a formal sense – it is a chance for them to explore and develop ideas in a non-judgmental setting.

- **<u>Pictorial representations</u>**

Children are encouraged to represent their learning in pictures whenever possible so that the abstract has meaning. Many of these pictures are specifically taught through the progression in written calculation. (See Strategies for Teaching Written Calculations)

Question Types

Children will be exposed to a variety of question types during the course of their maths lessons. They will become familiar with:

- True/False questions
- Conjecture
- Convince Me
- Spot the mistake
- Missing numbers and balancing equations
- = sign in different places
- Variety of vocabulary used as part of questions asked
- Generalisation
- Trial and Error
- Odd one out
- What's the Same? What's different?

Written Calculation Strategies

Progression in Written Calculation

These strategies for teaching calculation show progression as children move through different year groups. However, they may be used at any point during the children's education and so although methods build upon previous examples there is nothing to stop children revisiting a method in order to support their understanding and accuracy. For example, as numbers become larger they may revert to pictorial representations to support their learning; part-whole models may be used at any time to demonstrate how a problem may be solved; manipulatives should be accessible in every year group so that, at any time, they can be used to show how a problem can be solved.

Addition

The National Curriculum Expectations

(Curriculum 2014 Statutory Requirements)

EYFS – Pupils should be taught to:	
Birth to 11 months	• notice changes in number of objects/images or sounds in groups of up to 3
8 – 20 months	• has some understanding that things exist even when out of sight
16 – 26 months	• Begins to organise and categorise objects - sorting

22 – 36 months 30 – 50 months 40 – 60 months	• Knows that a group of things changes in quantity when something is added or taken away • Separates a group of 3 or 4 objects in different ways beginning to recognise that the total is still the same • Finds the total number of items in two groups by counting all of them • Says the number that is one more than a given number • Finds one more or one less from a group of up to five objects then ten objects • In practical activities and discussions begins to use the vocabulary involved in addition and subtraction **<u>Early Learning Goal</u> - Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.**
Year 1 – Pupils should be taught to:	
• Read, write and interpret mathematical statements involving addition (+) and equals (=) signs. • Represent and use number bonds and related subtraction facts within 20 • Add one-digit and two-digit numbers to 20, including zero • Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 9 = □ + 7.	
Year 2 – Pupils should be taught to:	
• Solve problems with addition:	

- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods

- Recall and use addition facts to 20 fluently, and derive and use related facts up to 100
- Add numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

Year 3 – Pupils should be taught to:

- Add numbers mentally, including:
  - a three-digit number and ones
  - a three-digit number and tens
  - a three-digit number and hundreds
- Add numbers with up to three digits, using formal written methods of column addition
- Estimate the answer to a calculation and use inverse operations to check answers
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.

Year 4 – Children should be taught to:

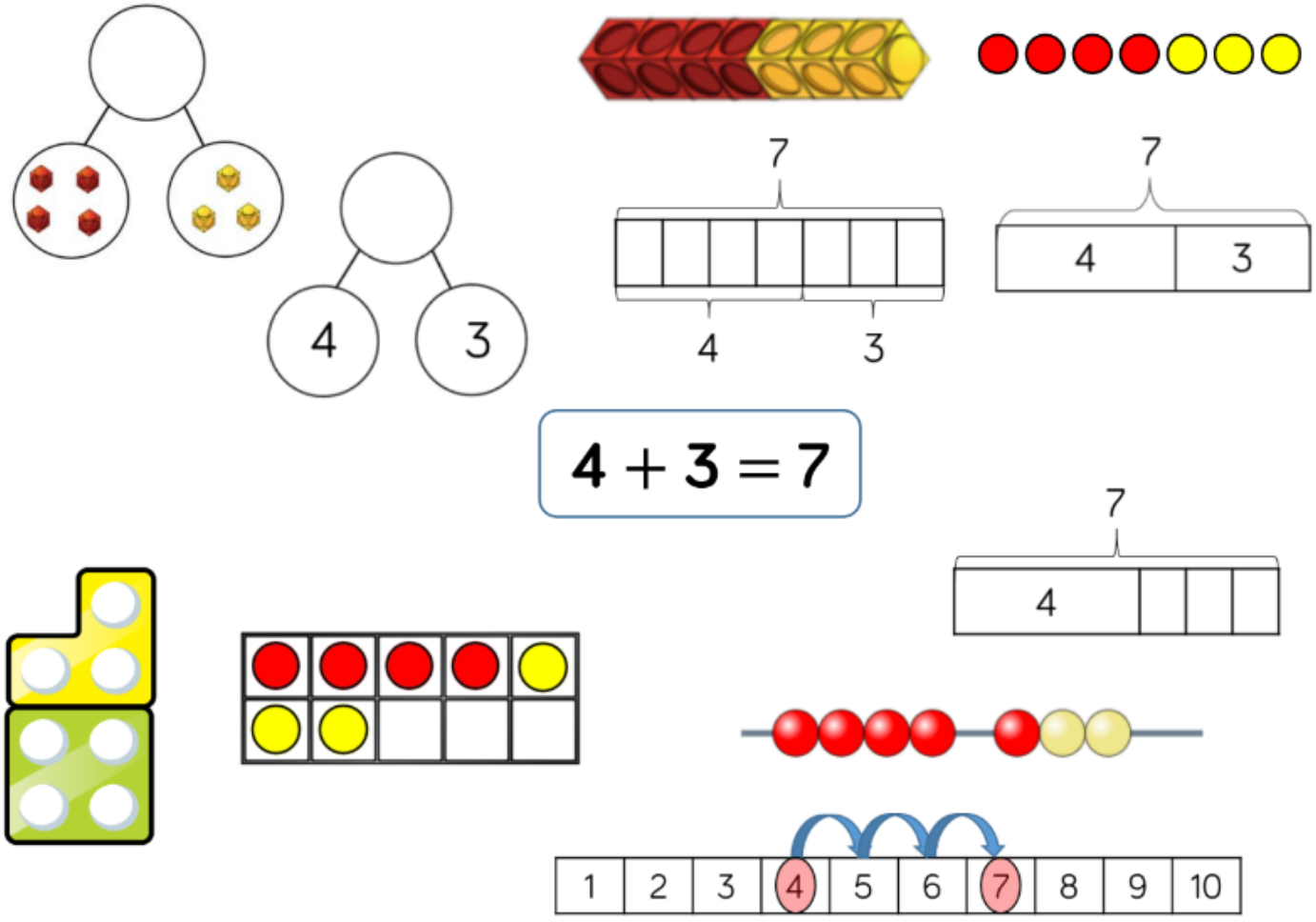
• Add numbers with up to 4 digits using the formal written methods of column addition where appropriate • Estimate and use inverse operations to check answers to a calculation • Solve addition two-step problems in contexts, deciding which operations and methods to use and **why**.
Year 5 – Children should be taught to:
• Add whole numbers with more than 4 digits, including using formal written methods (column addition) • Add numbers mentally with increasingly large numbers • Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • Solve addition multi-step problems in contexts, deciding which operations and methods to use and why.
Year 6 – Children should be taught to:
• Solve addition multi-step problems in contexts, deciding which operations and methods to use and why

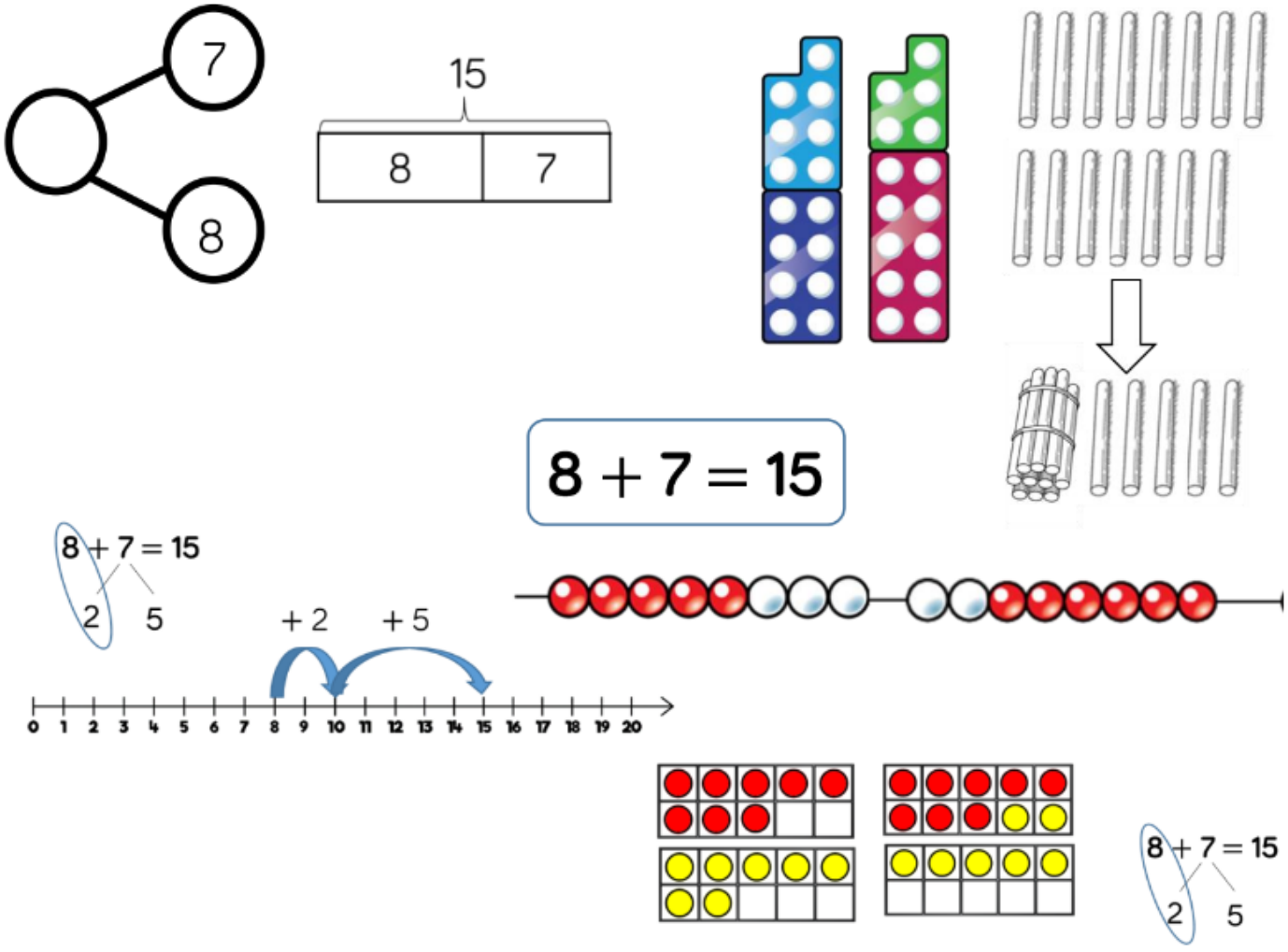
Strategies for Teaching

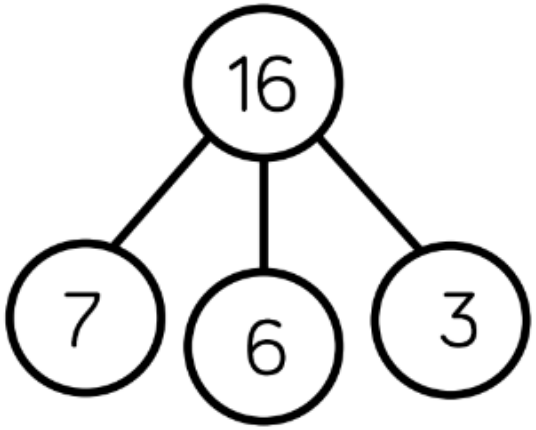
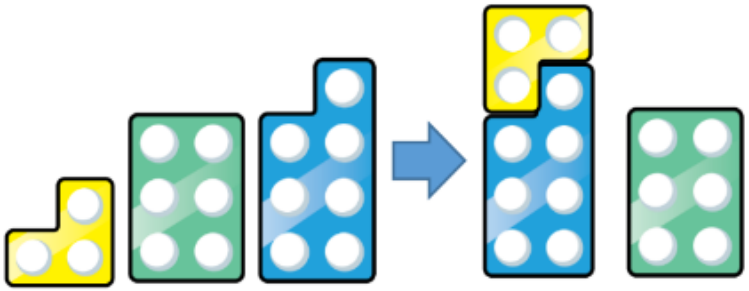
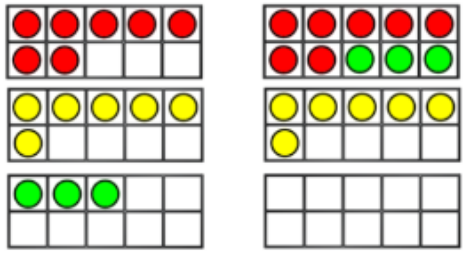
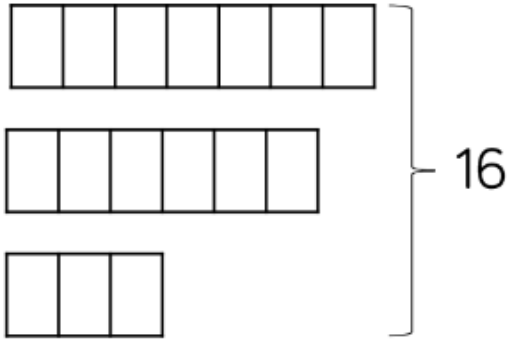
Addition

Skill	Year	Representations and models	
Add two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Add 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead strings (20) Number tracks Number lines (labelled) Straws
Add three 1-digit numbers	2	Part-whole model Bar model	Ten frames (within 20) Number shapes
Add 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square

Skill	Year	Representations and models	
Add two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column addition
Add with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with more than 4 digits	5	Part-whole model Bar model	Place value counters Column addition
Add with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column addition

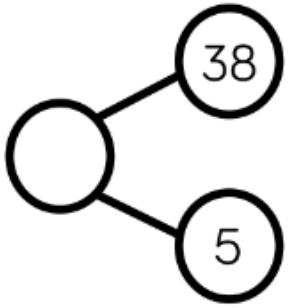
Skill: Add 1-digit numbers within 10	Year: 1
	When adding numbers to 10, children can explore both aggregation and augmentation. The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation. The combination bar model, ten frame, bead string and number track all support augmentation.

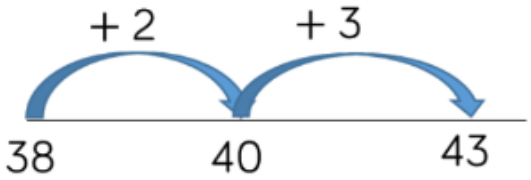
Skill: Add 1 and 2-digit numbers to 20	Year: 1/2
	When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.

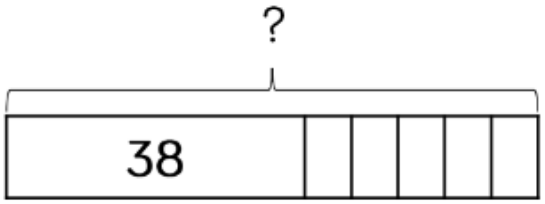
Skill: Add three 1-digit numbers	Year: 2
  $7 + 6 + 3 = 16$  $7 + 6 + 3 = 16$ 10 	When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Skill: Add 1-digit and 2-digit numbers to 100

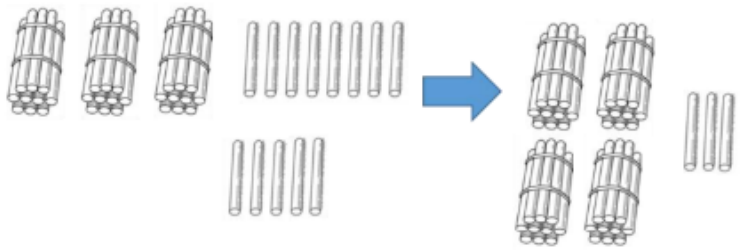
Year: 2/3







$$38 + 5 = 43$$



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

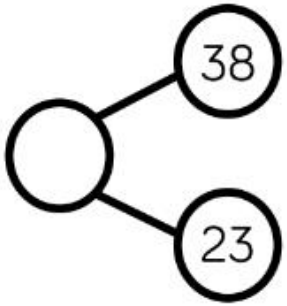
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

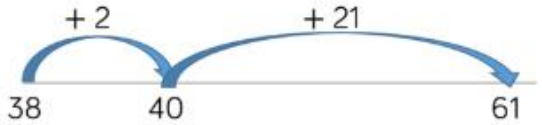
They should also apply their knowledge of number bonds to add more efficiently e.g. 8 + 5 = 13 so 38 + 5 = 43.

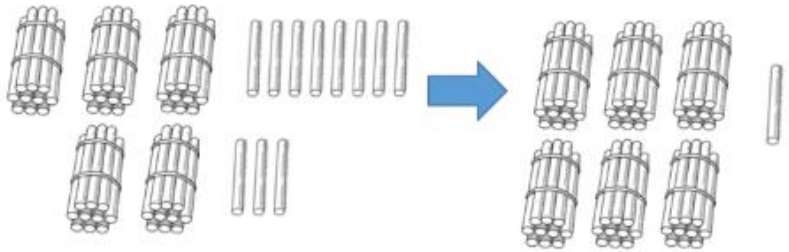
Hundred squares and straws can support children to find the number bond to 10.

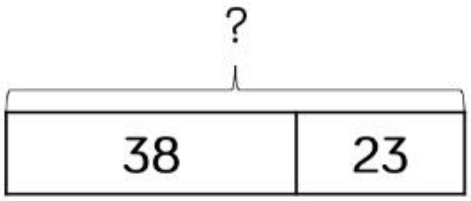
Skill: Add two 2-digit numbers to 100

Year: 2/3

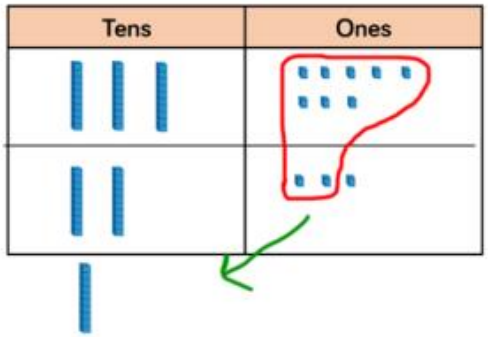




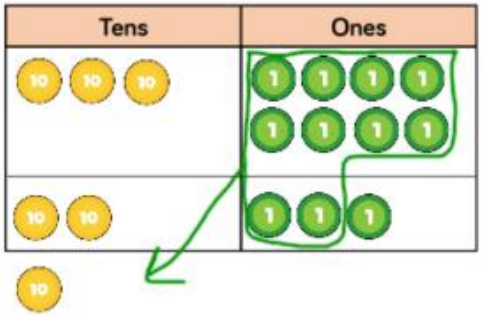




38 + 23 = 61

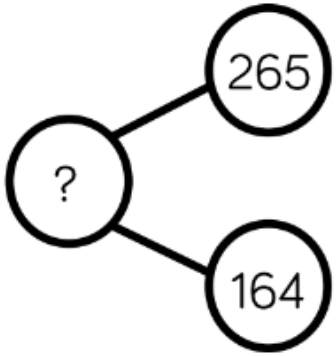
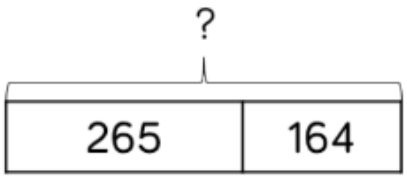
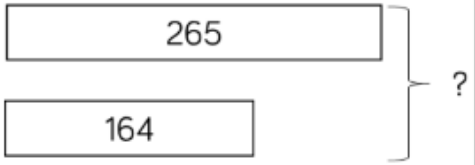
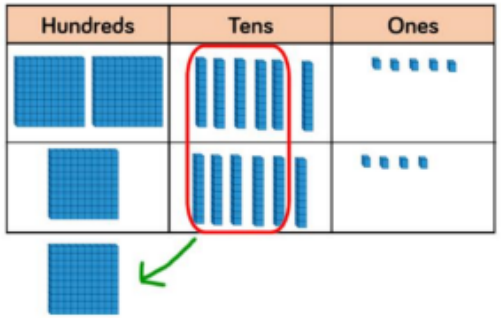
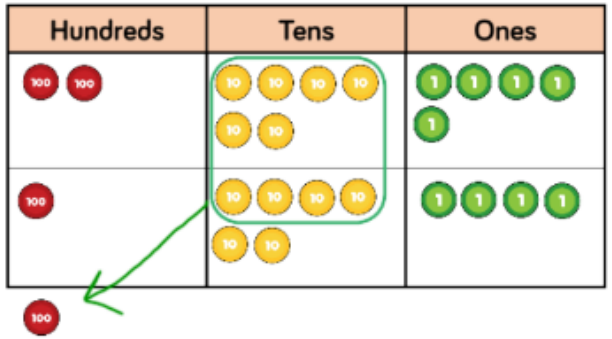


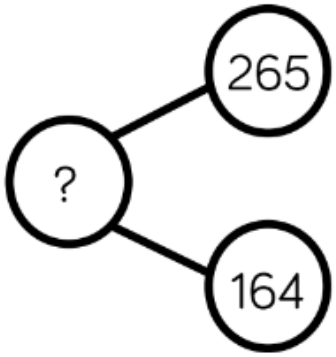
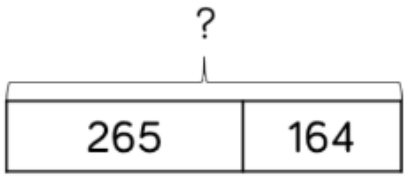
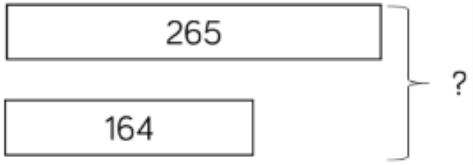
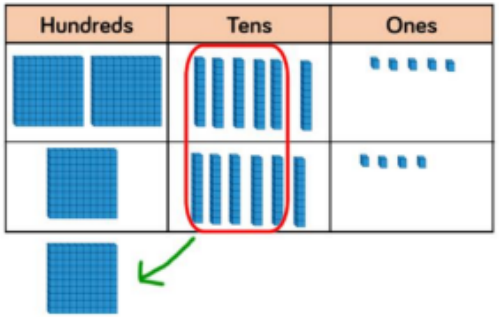
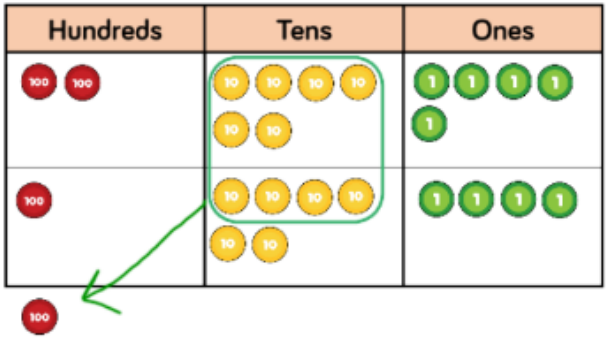
$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$



At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

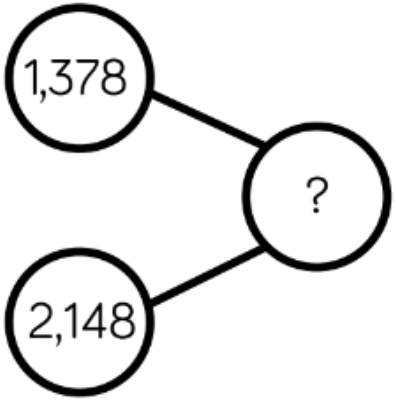
Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

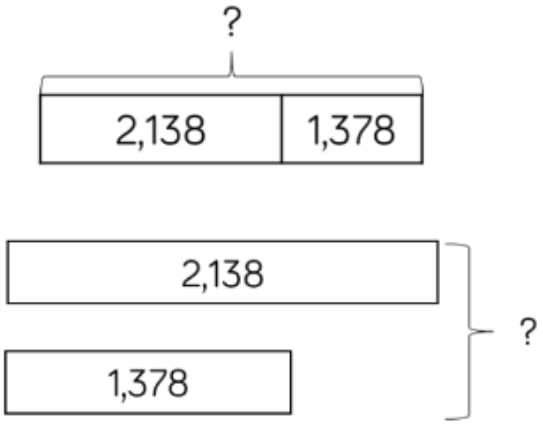
Skill: Add numbers with up to 3 digits	Year: 3
   **265 + 164 = 429**  265 + 164 429 1 	Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with up to 3 digits	Year: 3
   **265 + 164 = 429**  265 + 164 429 1 	Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with up to 4 digits

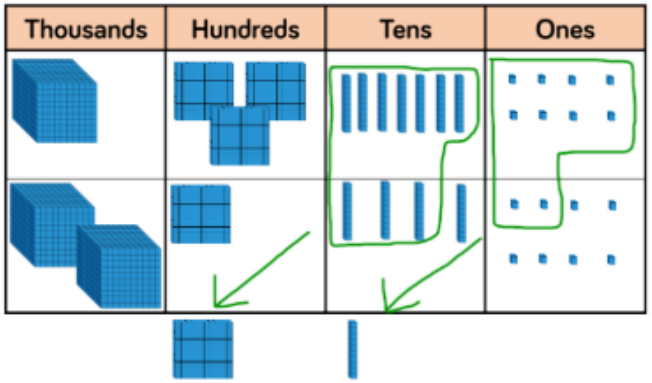
Year: 4

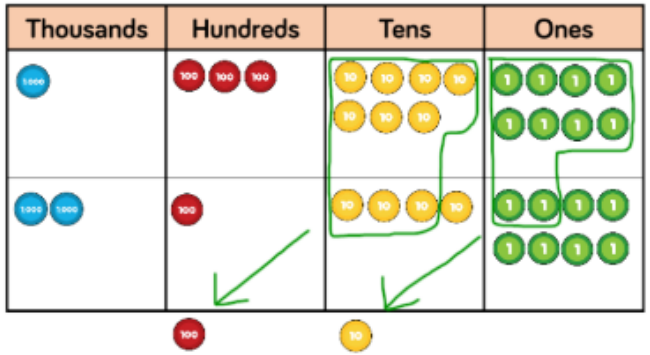




	1	3	7	8
+	2	1	4	8
	3	5	2	6
	1	1		

1,378 + 2,148 = 3,526





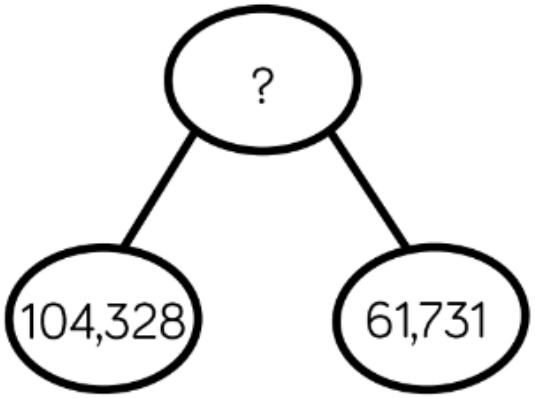
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

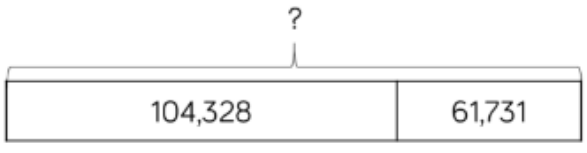
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

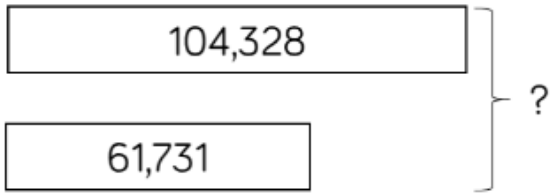
Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with more than 4 digits

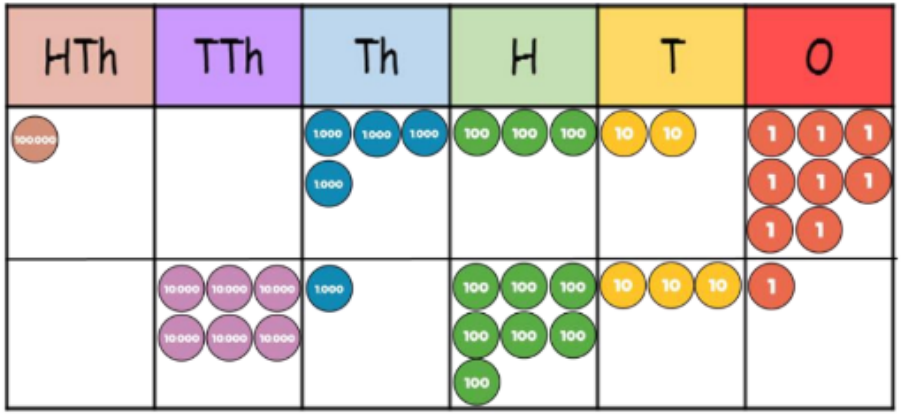
Year: 5/6







104,328 + 61,731 = 166,059

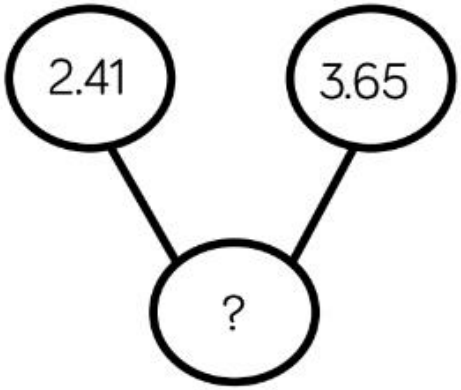
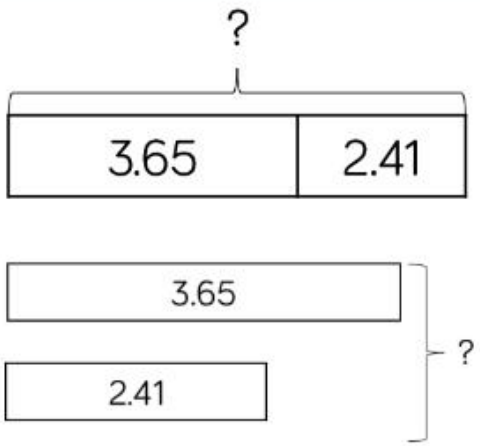
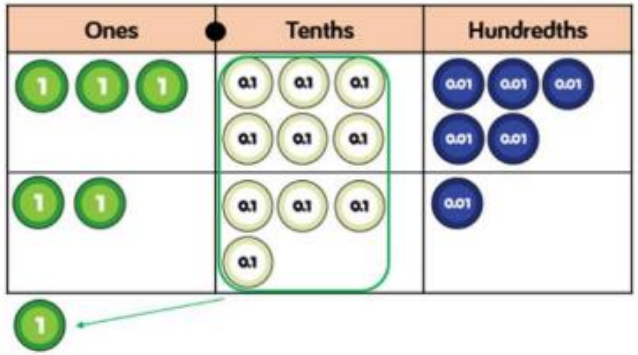
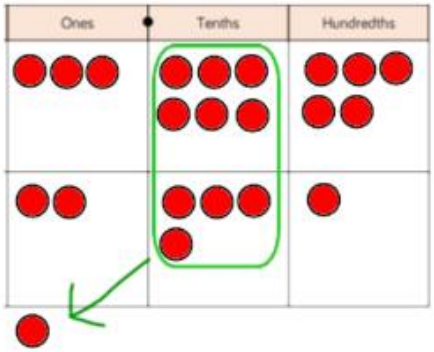


1	0	4	3	2	8
+	6	1	7	3	1
1	6	6	0	5	9

1

Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.

Skill: Add with up to 3 decimal places	Year: 5
  $\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$ **3.65 + 2.41 = 6.06**  	Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.

Subtraction

The National Curriculum Expectations

(Curriculum 2014 Statutory Requirements)

EYFS – Pupils should be taught to:	
Birth to 11 months	• notice changes in number of objects / images, sounds in groups of and up to 3
8 – 20 months	• has some understanding that things exist even when out of sight
16 – 26 months	• Begins to organise and categorise objects -sorting
22 – 36 months	• knows that a group of things changes in quantity when something is added or taken away
30 – 50 months	• separates a group of or 4 objects in different ways beginning to recognise that the total is still the same
40 – 60 months	• Understands subtraction as taking away objects from a group and counting on how many are left. • In practical activities and discussions begins to use the vocabulary involved in addition and subtraction **<u>Early Learning Goal</u> - Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer.**

Year 1 – Pupils should be taught to:
• read, write and interpret mathematical statements involving subtraction (-) and equals (=) signs • represent and use number bonds and related subtraction facts within 20 • subtract one-digit and two-digit numbers to 20, including zero • solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 9 = □ - 7.
Year 2 – Pupils should be taught to:
• solve problems with subtraction: - using concrete objects and pictorial representations, including those involving numbers, quantities and measures - applying their increasing knowledge of mental and written methods • recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100 • subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - subtracting three one-digit numbers • show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot • recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

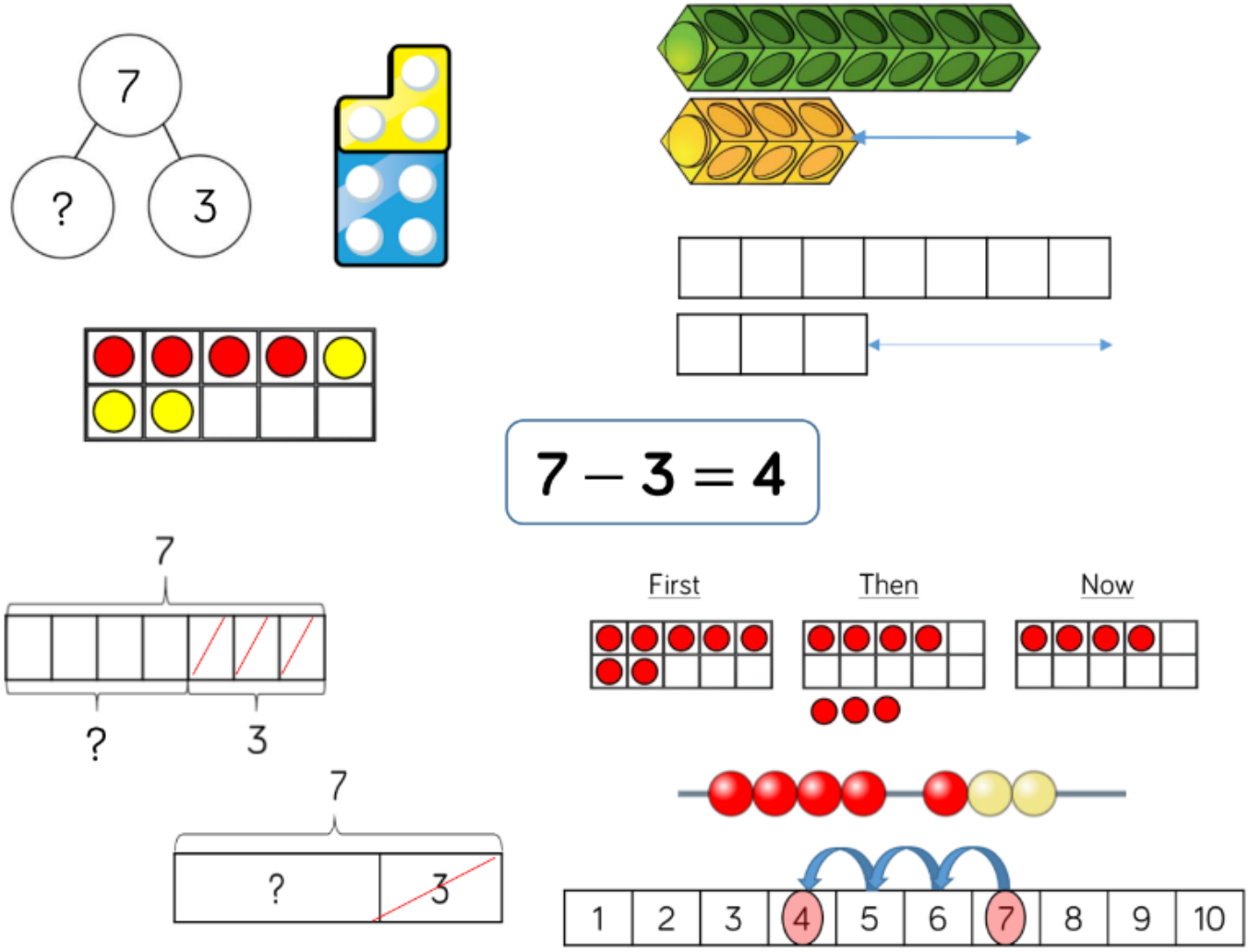
Year 3 – Pupils should be taught to:
• subtract numbers mentally, including: • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds • subtract numbers with up to three digits, using formal written methods of column subtraction • estimate the answer to a calculation and use inverse operations to check answers • solve problems, including missing number problems, using number facts, place value, and more complex subtraction.
Year 4 Pupils should be taught to:
• subtract with up to 4 digits using the formal written methods of column subtraction where appropriate • estimate and use inverse operations to check answers to a calculation • solve subtraction two-step problems in contexts, deciding which operations and methods to use and why.
Year 5 – Pupils should be taught to:
• subtract whole numbers with more than 4 digits, including using formal written methods (column subtraction) • subtract numbers mentally with increasingly large numbers • use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy • solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
Year 6 – Pupils should be taught to:
• solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Strategies for Teaching

Subtraction

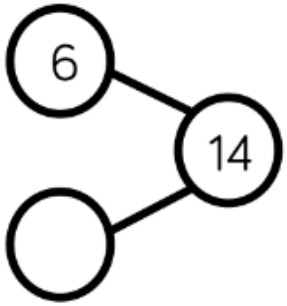
Skill	Year	Representations and models	
Subtract two 1-digit numbers to 10	1	Part-whole model Bar model Number shapes	Ten frames (within 10) Bead strings (10) Number tracks
Subtract 1 and 2-digit numbers to 20	1	Part-whole model Bar model Number shapes Ten frames (within 20)	Bead string (20) Number tracks Number lines (labelled) Straws
Subtract 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Straws Hundred square
Subtract two 2-digit numbers	2	Part-whole model Bar model Number lines (blank) Straws	Base 10 Place value counters Column subtraction

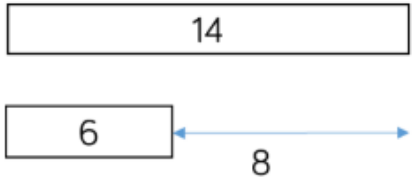
Skill	Year	Representations and models	
Subtract with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column subtraction
Subtract with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column subtraction
Subtract with more than 4 digits	5	Part-whole model Bar model	Place value counters Column subtraction
Subtract with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column subtraction

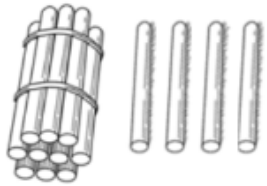
Skill: Subtract 1-digit numbers within 10	Year: 1
	Part-whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with two bars can support finding the difference.

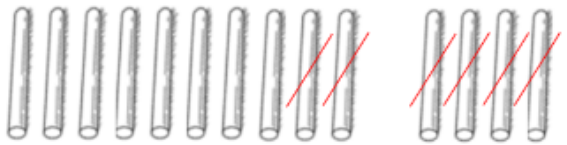
Skill: Subtract 1 and 2-digit numbers to 20

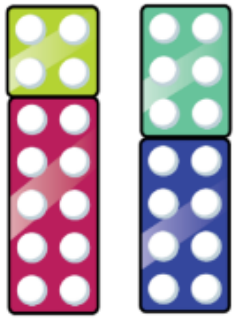
Year: 1/2



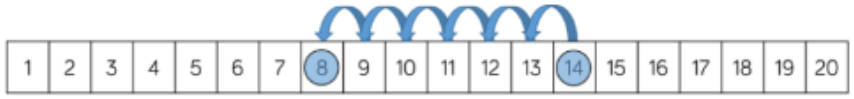


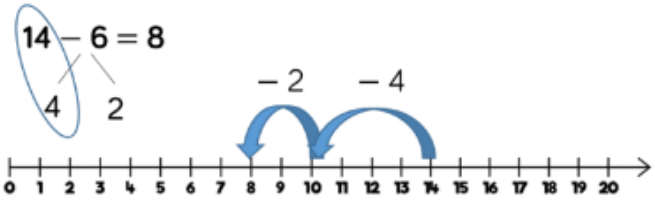


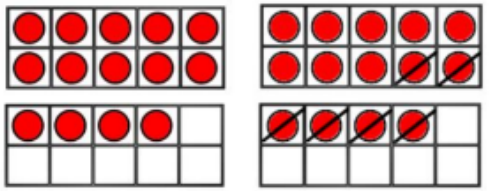


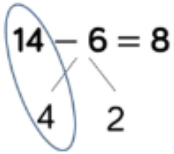


14 − 6 = 8



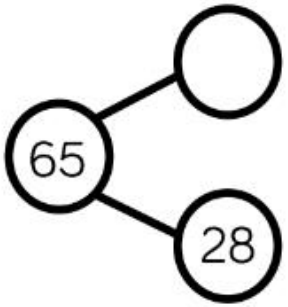
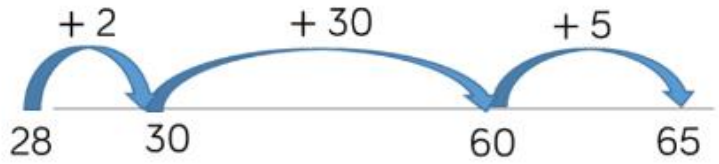
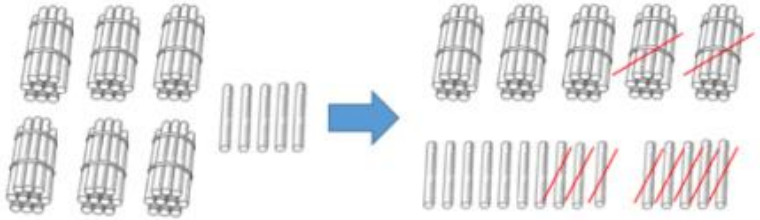
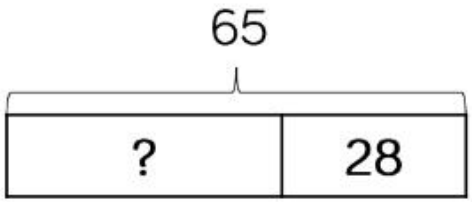
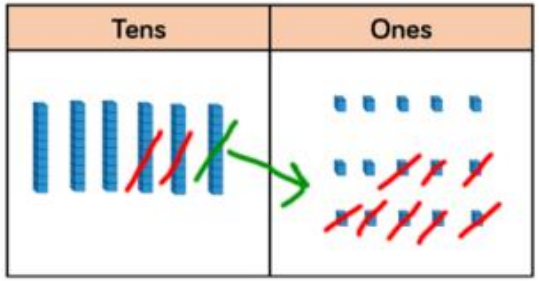
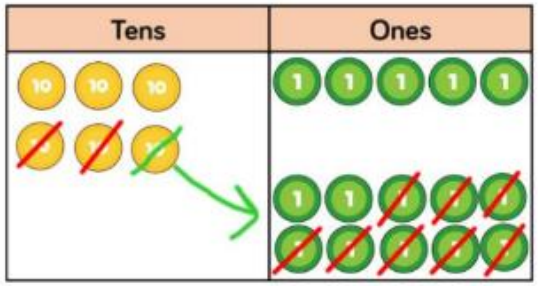


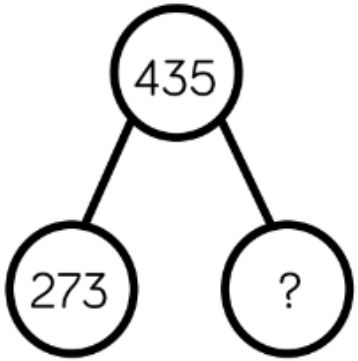
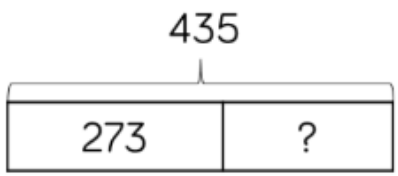
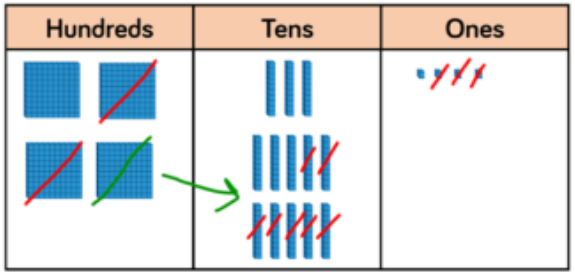
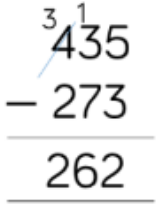
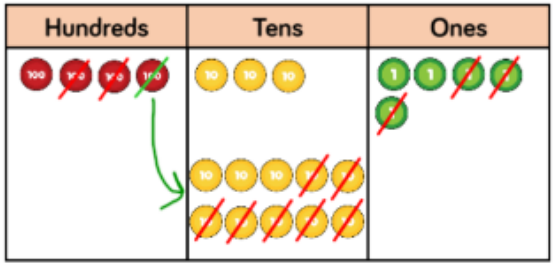




When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten.

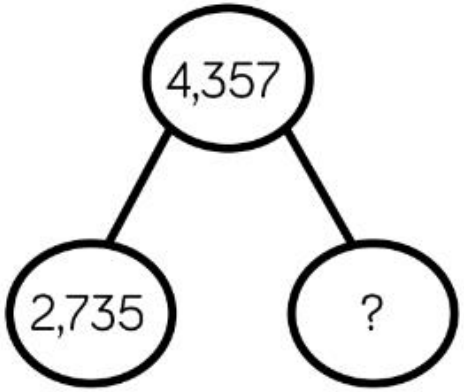
Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.

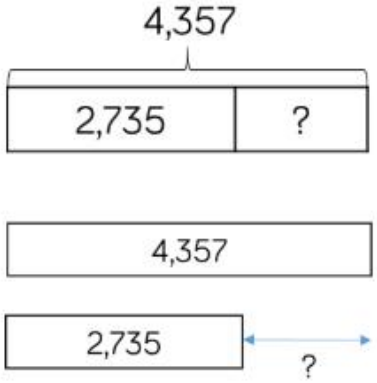
Skill: Subtract 1 and 2-digit numbers to 100	Year: 2
    $65 - 28 = 37$  $\begin{array}{r} {}^{5}\not{6}\,{}^{1}\not{5} \\ -\ 2\ 8 \\ \hline 3\ 7 \end{array}$ 	At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient. Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Subtract numbers with up to 3 digits	Year: 3
   **435 − 273 = 262**   	Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with up to 4 digits

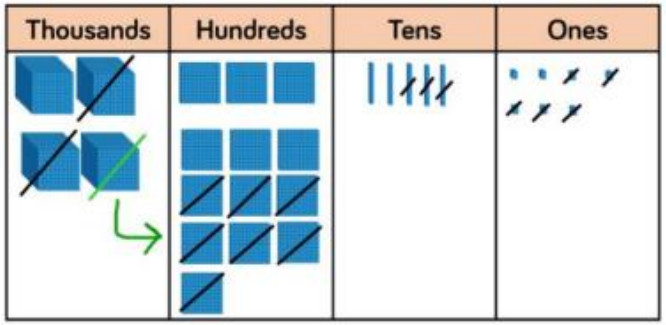
Year: 4

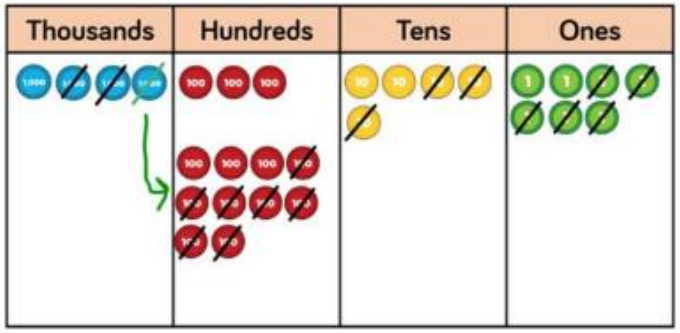




$$\begin{array}{r} {}^{3}\cancel{4}\,{}^{1}357 \\ -\ 2735 \\ \hline 1622 \end{array}$$

$$4{,}357 - 2{,}735 = 1{,}622$$





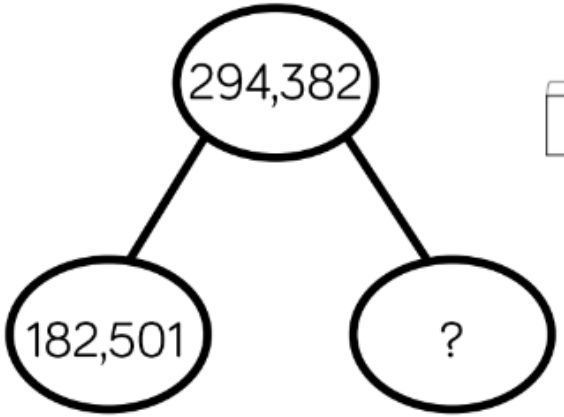
Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits.

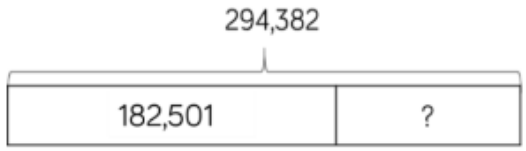
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

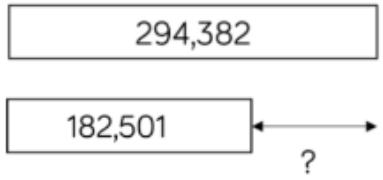
Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with more than 4 digits

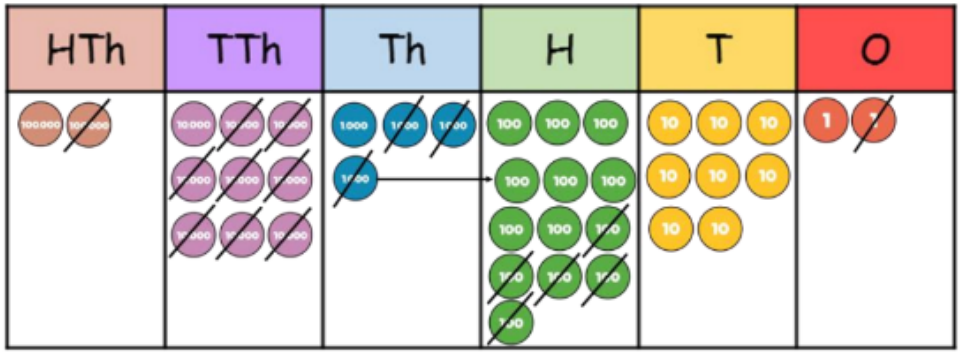
Year: 5/6







294,382 − 182,501 = 111,881



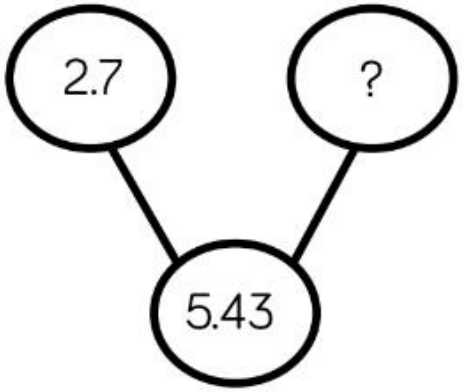
	2	9	~~3~~ 4	1 3	8	2
−	1	8	2	5	0	1
	1	1	1	8	8	1

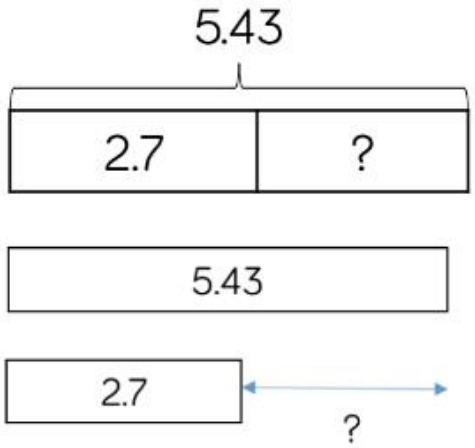
Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

Skill: Subtract with up to 3 decimal places

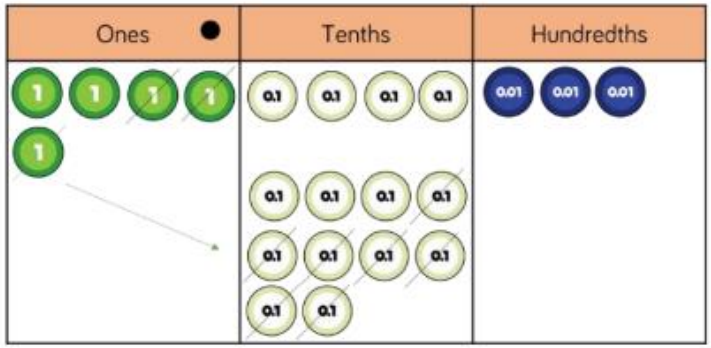
Year: 5

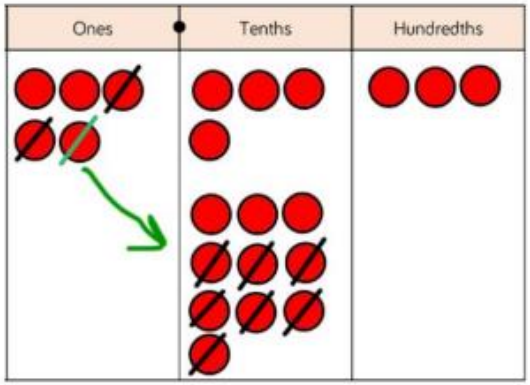




$$\begin{array}{r} {}^{4}\cancel{5}.{}^{1}43 \\ -\ 2.7 \\ \hline 2.73 \end{array}$$

5.43 − 2.7 = 2.73





Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative – numbers can be added in any order.

Complement – in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference – the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange – Change a number or expression for another of an equal value.

Minuend – A quantity or number from which another is subtracted.

Partitioning – Splitting a number into its component parts.

Reduction – Subtraction as take away.

Subitise – Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total – The aggregate or the sum found by addition.

Multiplication

The National Curriculum Expectations

(Curriculum 2014 Statutory Requirements)

EYFS – Pupils should be taught to:
Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.
Year 1 Pupils should be taught to:
• solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Year 2 Pupils should be taught to:
• recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs • show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

• solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
Year 3 Pupils should be taught to:
• recall and use multiplication facts for the 3, 4 and 8 multiplication tables • write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to written methods • solve problems involving missing number problems involving multiplication including positive number scaling problems and correspondence problems where n objects are connected to m objects.
Year 4 Pupils should be taught to:
• recall and use multiplication facts for multiplication tables up to 12 x 12 • use place value, known and derived facts to multiply mentally, including: x0 x1 and multiplying together three numbers • recognise and use factor pairs and commutativity in mental calculations • multiply two-digit and three-digit numbers by a one-digit number using formal written layout • solve problems involving multiplying, including the distributive law to multiply two-digit numbers by one-digit including positive number scaling problems and correspondence problems where n objects are connected to m objects.
Year 5 Pupils should be taught to:
• identify multiples and factors: all factor pairs of a number, common factors of two numbers, establish whether a number up to 100 is prime and recall prime numbers up to 19 • multiply numbers up to four digits by a one- or two-digit number using a formal written method

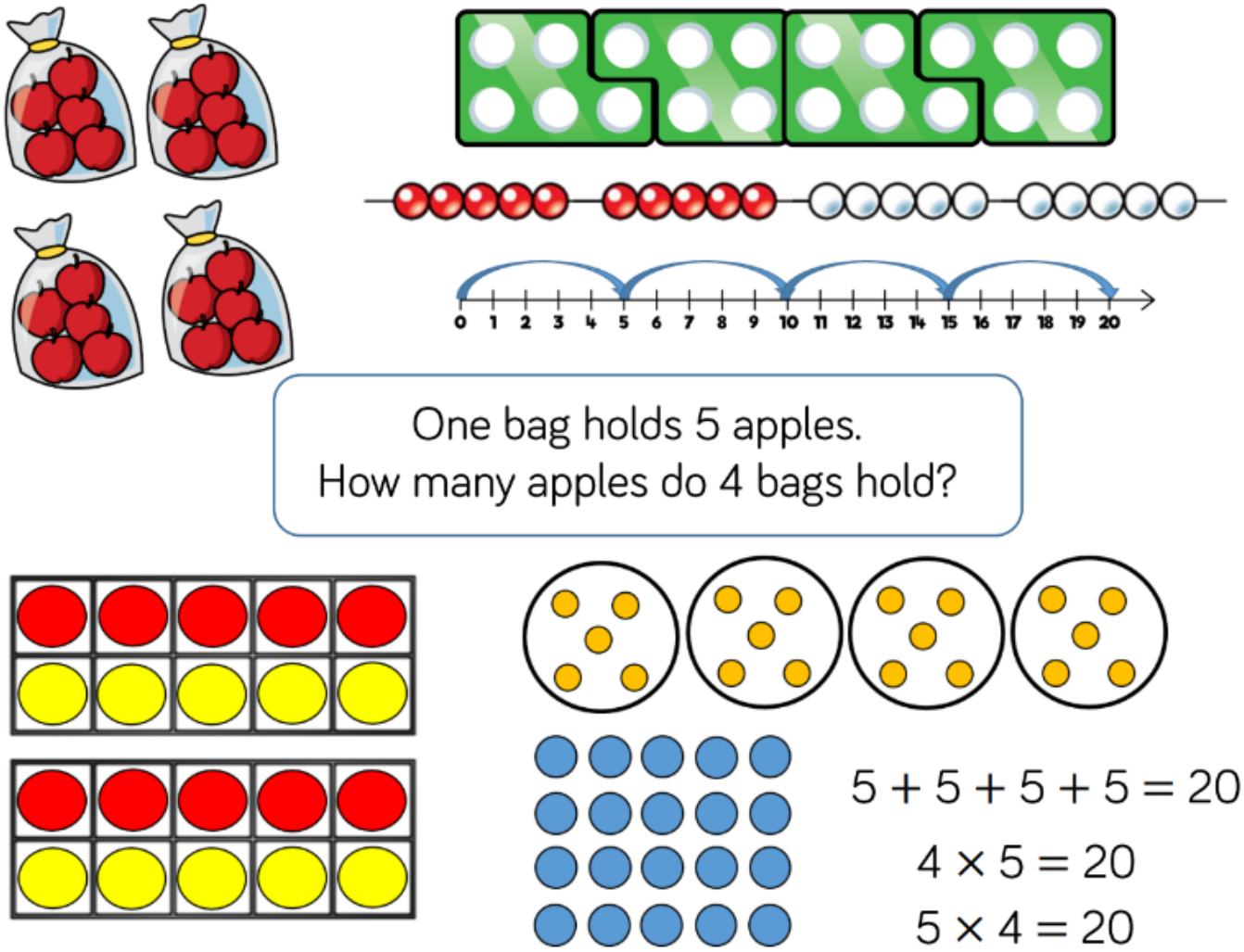
• multiply whole numbers and those involving decimals by 10, 100 and 1000. •
Year 6 Pupils should be taught to:
• identify multi-digit numbers up to 4 digits by a two-digit number using formal, long multiplication • identify common factors, common multiples and common prime numbers • use their knowledge of the order of operations to carry out calculations involving the four operations

Strategies for Teaching

Multiplication

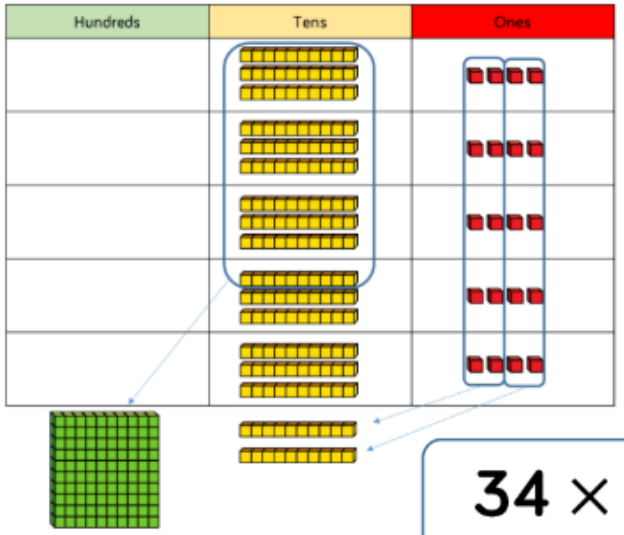
Skill	Year	Representations and models	
Solve one-step problems with multiplication	1/2	Bar model Number shapes Counters	Ten frames Bead strings Number lines
Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Short written method Expanded written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method
Multiply 4-digit by 1-digit numbers	5	Place value counters	Short written method

Skill	Year	Representations and models	
Multiply 2-digit by 2-digit numbers	5	Place value counters Base 10	Short written method Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method	

Skill: Solve 1-step problems using multiplication	Year: 1/2
	Children represent multiplication as repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2, children are introduced to the multiplication symbol.

Skill: Multiply 2-digit numbers by 1-digit numbers

Year: 3/4



	H	T	O	
		3	4	
×			5	
		2	0	(5 × 4)
+	1	5	0	(5 × 30)
	1	7	0	

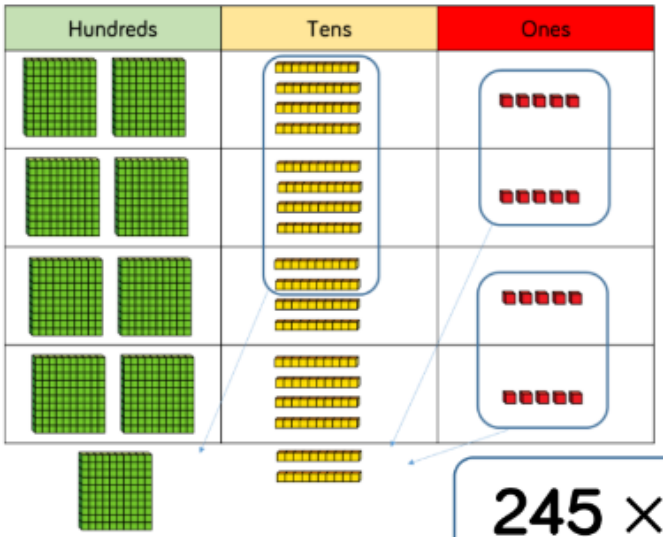
$$34 \times 5 = 170$$

	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		

Teachers may decide to first look at the expanded column method before moving on to the short multiplication method.
The place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

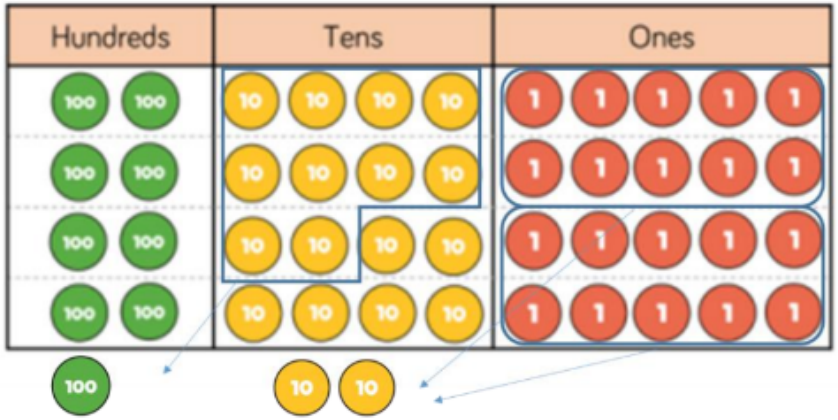
Skill: Multiply 3-digit numbers by 1-digit numbers

Year: 3/4



	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

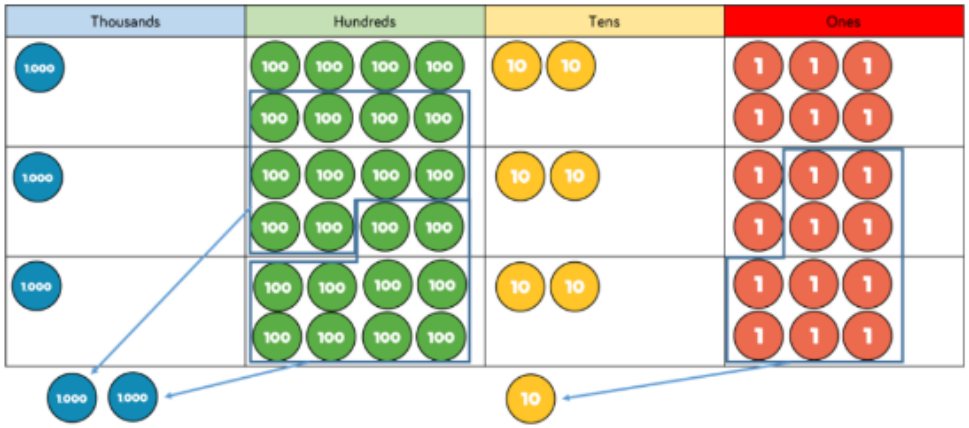
$$245 \times 4 = 980$$



When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method.
Base 10 and place value counters continue to support the understanding of the written method.
Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5



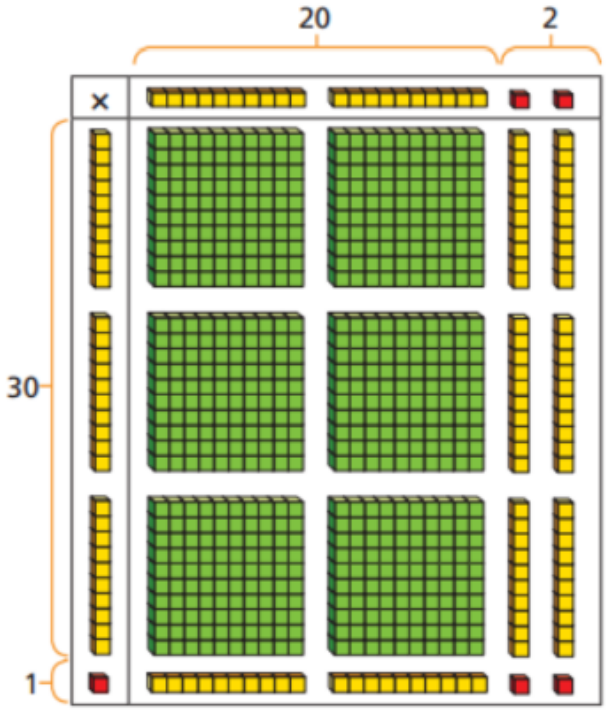
$$1{,}826 \times 3 = 5{,}478$$

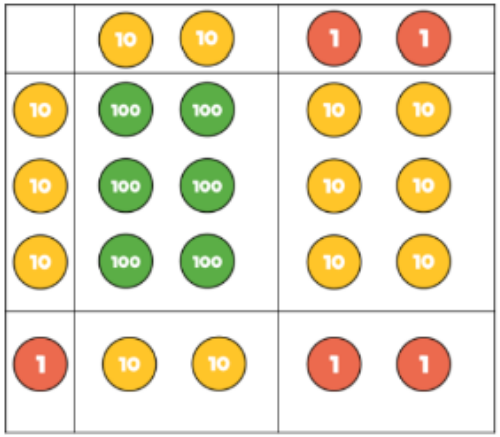
	Th	H	T	O
	1	8	2	6
×				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method.
If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 2-digit numbers by 2-digit numbers

Year: 5





×	20	2
30	600	60
1	20	2

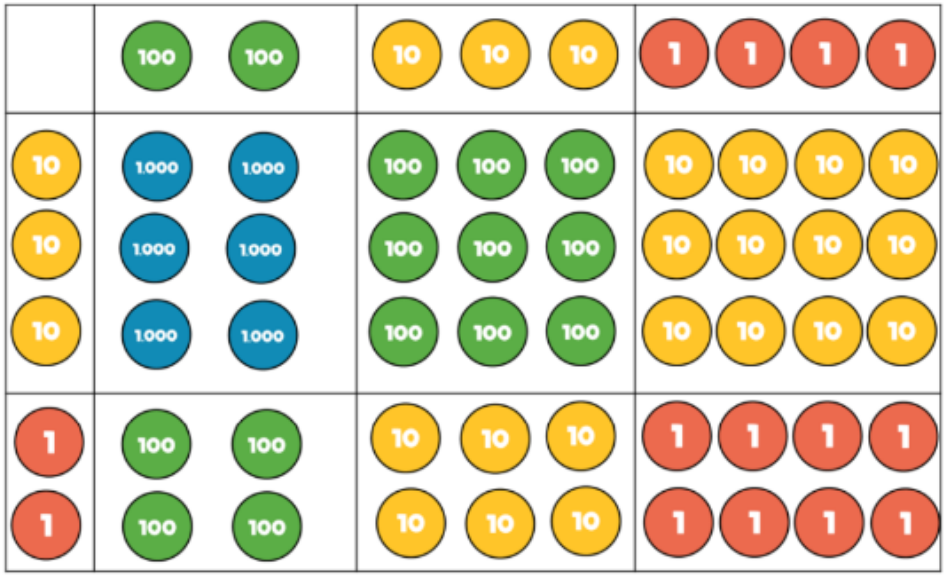
	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

$22 \times 31 = 682$

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10.
The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5



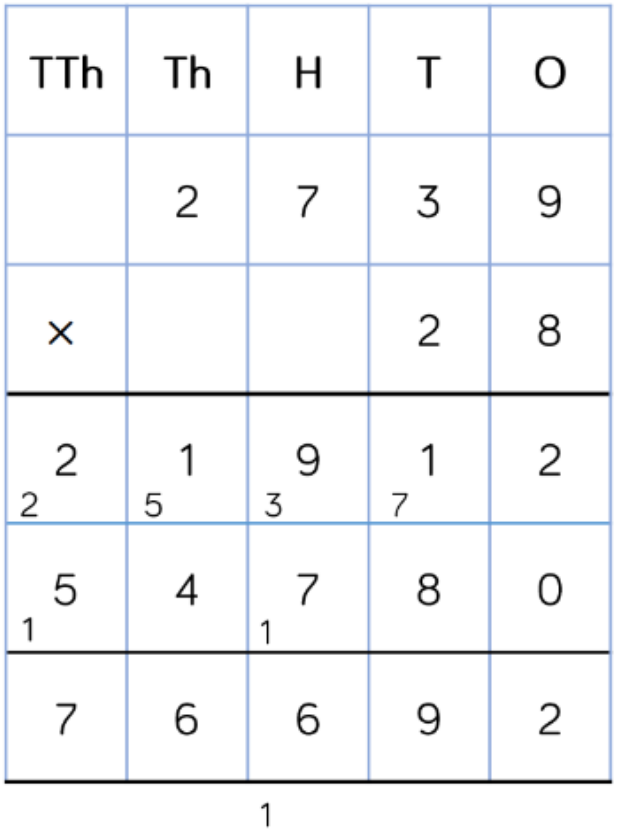
Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$^{1}7$	$^{1}0$	2	0
7	4	8	8

×	200	30	4
30	6,000	900	120
2	400	60	8

$234 \times 32 = 7{,}488$

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

Encourage children to move towards the formal written method, seeing the links with the grid method.

Skill: Multiply 4-digit numbers by 2-digit numbers	Year: 5/6
 $2{,}739 \times 28 = 76{,}692$	When multiplying 4-digits by 2-digits, children should be confident in the written method. If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method. Consider where exchanged digits are placed and make sure this is consistent.

Division

The National Curriculum Expectations

(Curriculum 2014 Statutory Requirements)

EYFS – Pupils should be taught to:
Early Learning Goal Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.
Year 1 – Pupils should be taught to:
• solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
Year 2 – Pupils should be taught to:
• recall and use multiplication and division facts for the 2, 3, 5 and 10 multiplication tables, including recognising odd and even numbers • calculate mathematical statements for division within the multiplication tables and write them using the signs ÷ and = • show that multiplication of two numbers is commutative but division is not • solve problems involving division using materials, arrays, repeated addition, mental methods and division facts, including problems in contexts.
Year 3 – Pupils should be taught to:

• recall and use multiplication and division facts for the 3, 4 and 8 x tables • write and calculate mathematical statements for division using the multiplication tables they know, including 2-digit divided by 1-digit using mental and progressing to formal written methods • solve problems, involving missing number problems, involving division, including positive number scaling problems and correspondence problems where n objects are connected to m objects.
Year 4 – Pupils should be taught to:
• recall multiplication and division facts up to 12 x 12 • use place value, known and derived facts to divide mentally, including dividing by 1 • solve problems involving dividing a three-digit number by one-digit number using a formal layout
Year 5 – Pupils should be taught to:
• identify multiples and factors, including finding all factor pairs of a number, common factors of two numbers, know and use the vocabulary of prime numbers and establish whether a number up to 100 is prime • multiply and divide numbers mentally drawing on known facts • divide numbers up to 4 digits by a one-digit number using a written method and interpret remainders appropriately for the context • divide whole numbers and those involving decimals by 10, 100 and 1000.
Year 6 – Pupils should be taught to:
• divide numbers up to 4 digits by a two-digit number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context. • divide numbers up to 4 digits by a two-digit number using the formal written method of short division as appropriate.

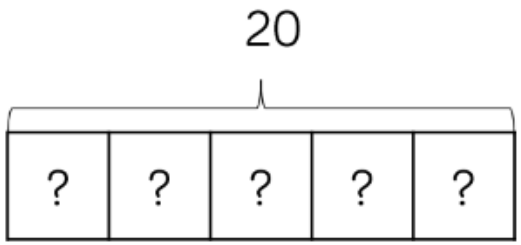
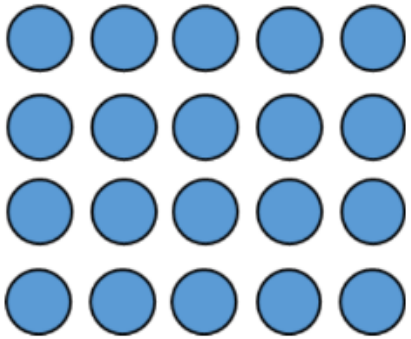
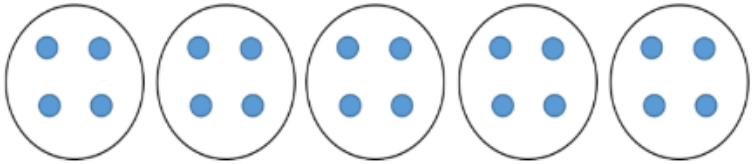
Strategies for Teaching

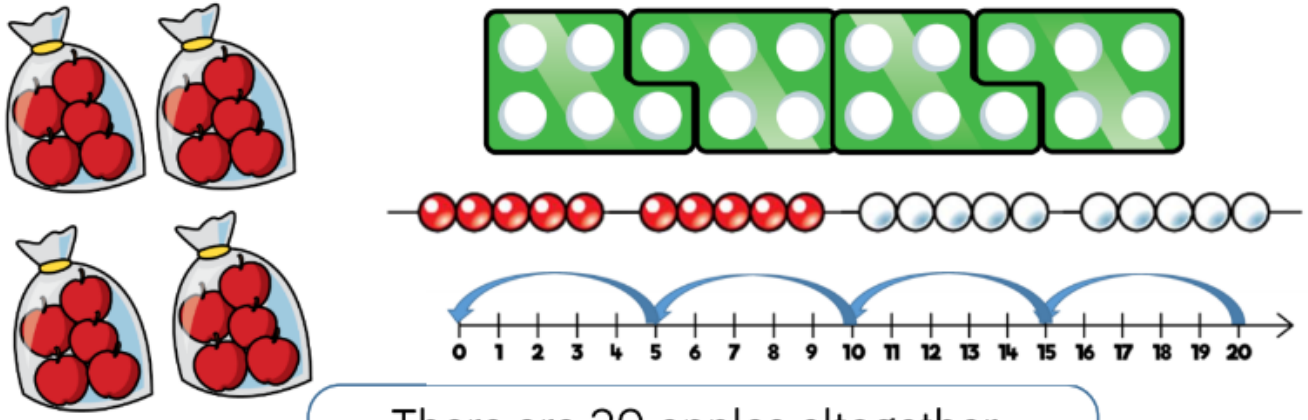
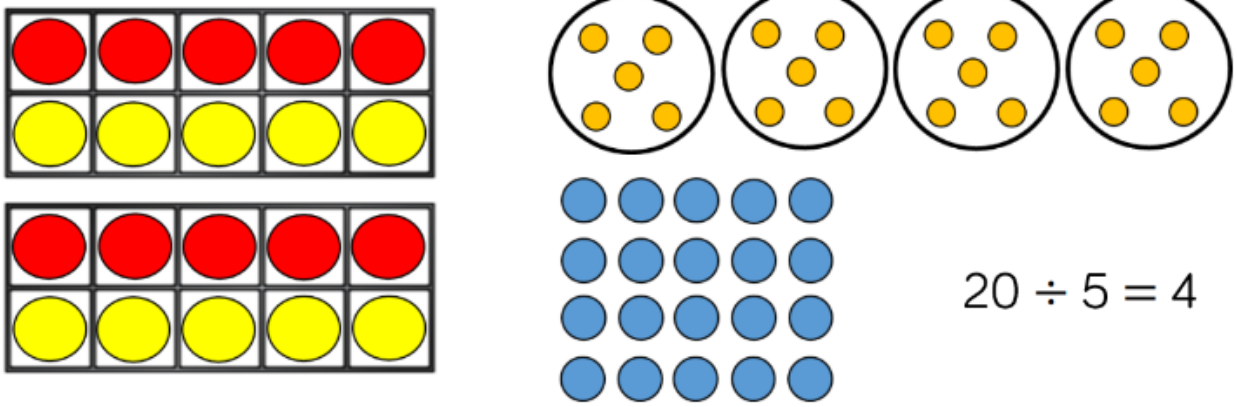
Division

Skill	Year	Representations and models	
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters
Solve one-step problems with division (grouping)	1/2	Real life objects Number shapes Bead strings Ten frames	Number lines Arrays Counters
Divide 2-digits by 1-digit (no exchange sharing)	3	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Straws Base 10 Bar model	Place value counters Part-whole model

Skill	Year	Representations and models	
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Straws Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

Skill	Year	Representations and models	
Divide 4-digits by 1-digit (grouping)	5	Place value counters Counters	Place value grid Written short division
Divide multi-digits by 2-digits (short division)	6	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	6	Written long division	List of multiples

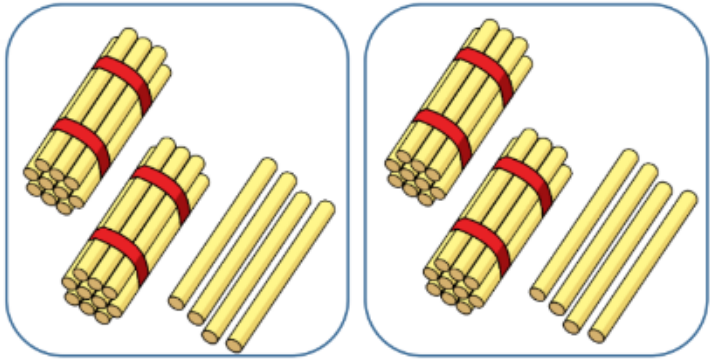
Skill: Solve 1-step problems using multiplication (sharing)	Year: 1/2
  There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?   $20 \div 5 = 4$	Children solve problems by sharing amounts into equal groups. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally. In Year 2, children are introduced to the division symbol.

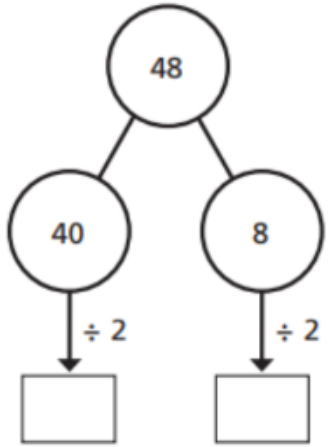
Skill: Solve 1-step problems using division (grouping)	Year: 1/2
 There are 20 apples altogether. They are put in bags of 5. How many bags are there?  $20 \div 5 = 4$	Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

Skill: Divide 2-digits by 1-digit (sharing with no exchange)

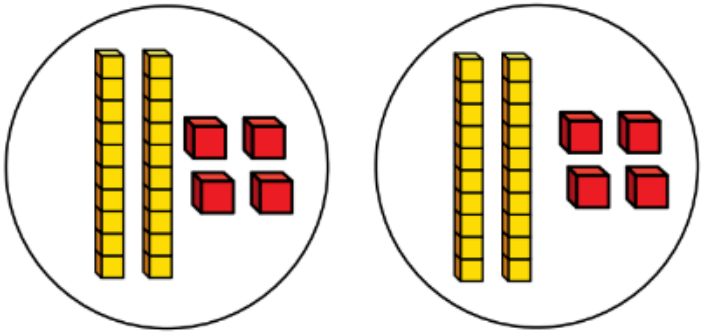
Year: 1/2

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1





$$48 \div 2 = 24$$



When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

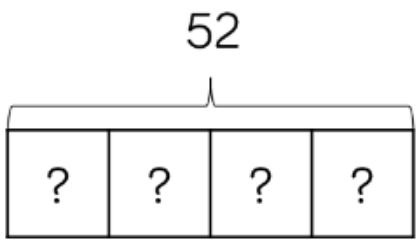
Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

Part-whole models can provide children with a clear written method that matches the concrete representation.

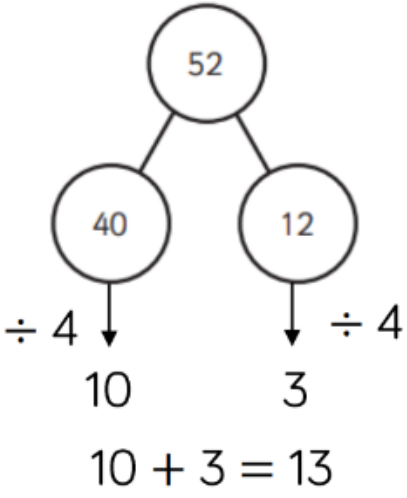
Skill: Divide 2-digits by 1-digit (sharing with exchange)

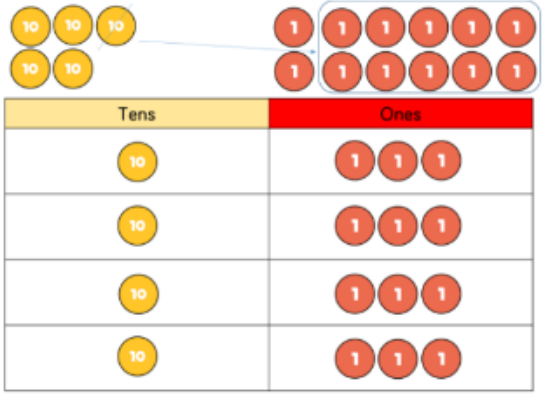
Year: 3/4

Tens	Ones



52 ÷ 4 = 13





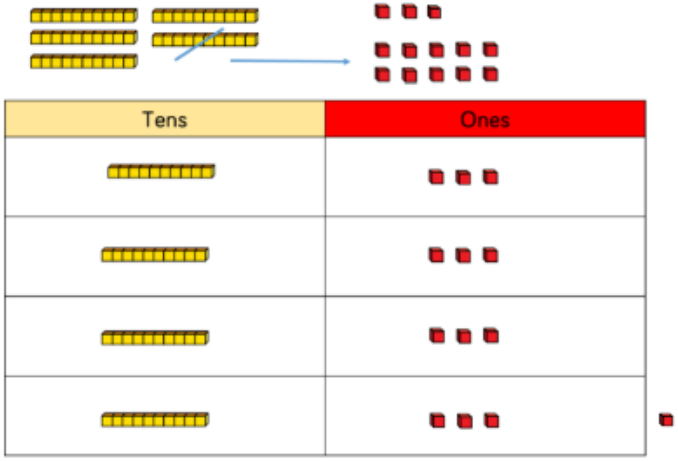
Tens	Ones
10	1 1 1
10	1 1 1
10	1 1 1
10	1 1 1

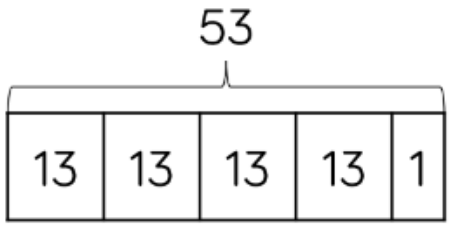
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones.
Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

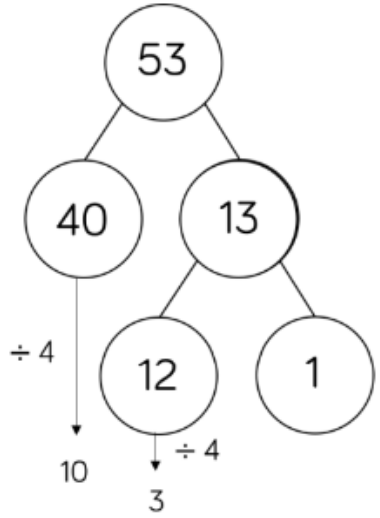
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

Year: 3/4





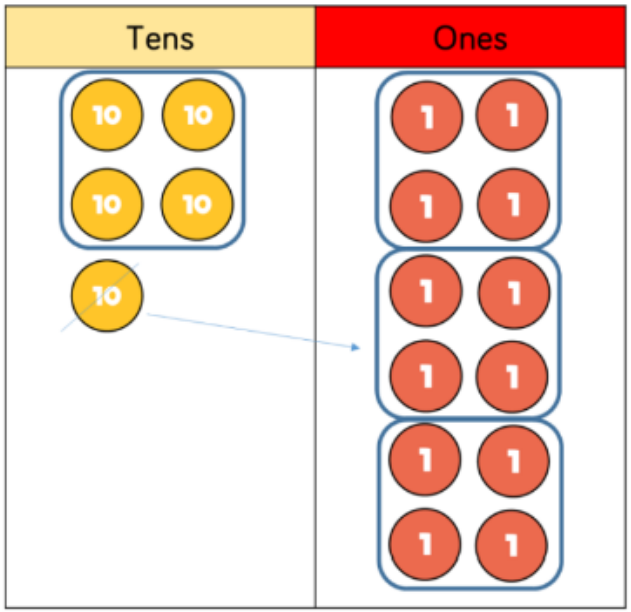


53 ÷ 4 = 13 r1

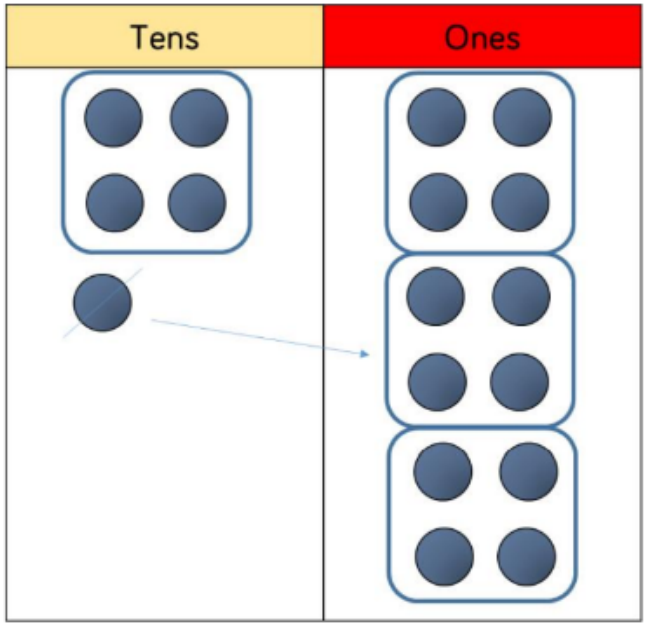
When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones.
Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made.
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 2-digits by 1-digit (grouping)

Year: 4/5



		1	3	
	4	5	1 2	



$$52 \div 4 = 13$$

When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

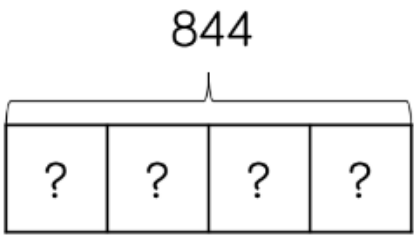
Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

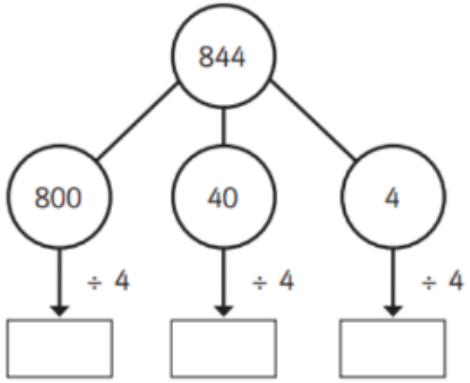
Skill: Divide 3-digits by 1-digit (sharing)

Year: 4

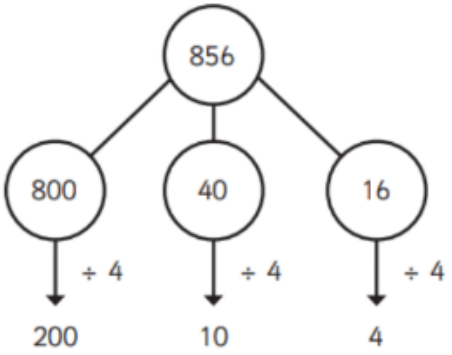
844 ÷ 4 = 211

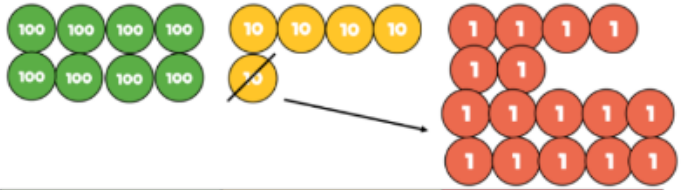


H	T	O
100 100	10	1
100 100	10	1
100 100	10	1
100 100	10	1



844 ÷ 4 = 211



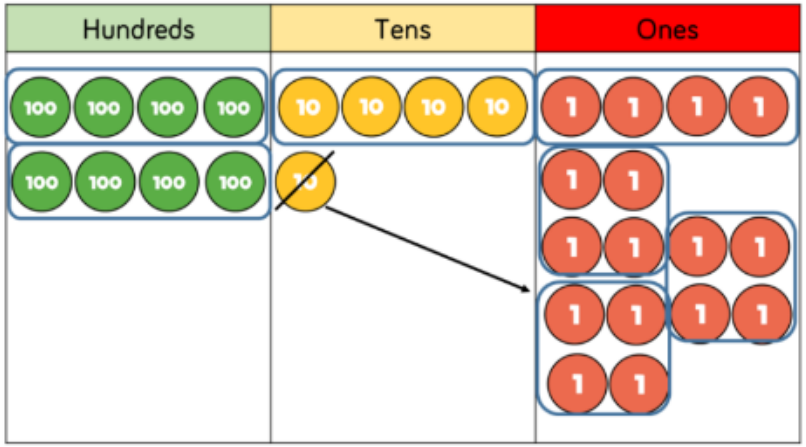


Hundreds	Tens	Ones
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1
100 100	10	1 1 1 1

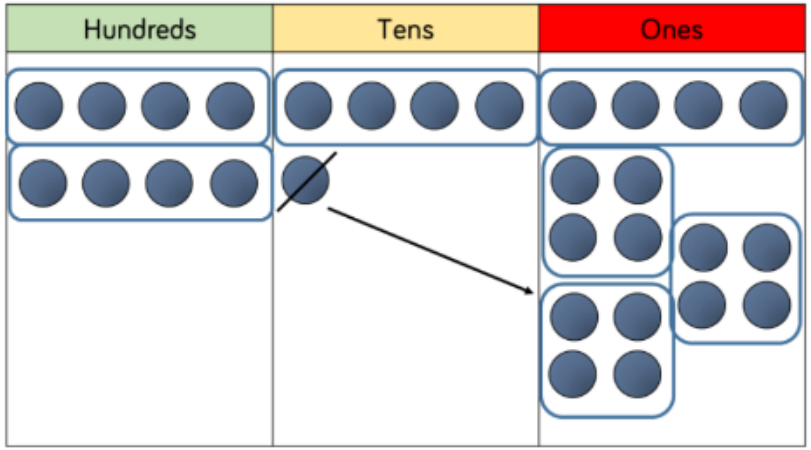
Children can continue to use place value counters to share 3-digit numbers into equal groups.
Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows.
This method can also help to highlight remainders.
Flexible partitioning in a part-whole model supports this method.

Skill: Divide 3-digits by 1-digit (grouping)

Year: 5



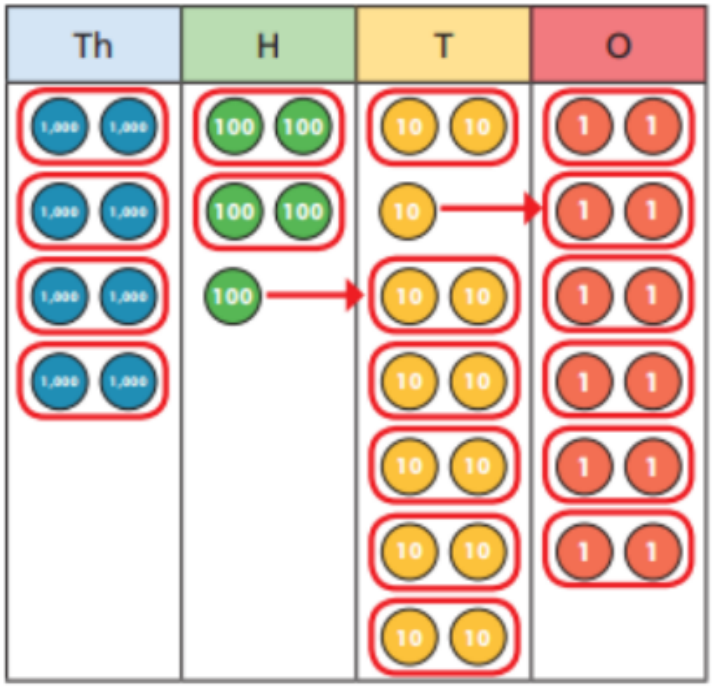
		2	1	4
	4	8	5	$^{1}6$



$856 \div 4 = 214$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide 4-digits by 1-digit (grouping)	Year: 5
 $8{,}532 \div 2 = 4{,}266$	Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method. Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

	4	2	6	6
2	8	5	$^{1}3$	$^{1}2$

Skill: Divide multi digits by 2-digits (short division)	Year: 6
	When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.

		0	3	6
	12	4	$^{4}3$	$^{7}2$

$432 \div 12 = 36$

$7{,}335 \div 15 = 489$

	0	4	8	9
15	7	$^{7}3$	$^{13}3$	$^{13}5$

15	30	45	60	75	90	105	120	135	150

Skill: Divide multi-digits by 2-digits (long division)	Year: 6

		0	3	6
1	2	4	3	2
	–	3	6	0
			7	2
	–		7	2
				0

(×30)

(×6)

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

$432 \div 12 = 36$

$7{,}335 \div 15 = 489$

	0	4	8	9	
15	7	3	3	5	
–	6	0	0	0	(×400
	1	3	3	5	
–	1	2	0	0	(×80)
		1	3	5	
–		1	3	5	(×9)
				0	

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Children can also divide by 2-digit numbers using long division.

Children can write out multiples to support their calculations with larger remainders.

Children will also solve problems with remainders where the quotient can be rounded as appropriate.

Skill: Divide multi digits by 2-digits (long division)	Year: 6
$372 \div 15 = 24\ r12$ $372 \div 15 = 24\frac{4}{5}$	When a remainder is left at the end of a calculation, children can either leave it as a remainder or convert it to a fraction. This will depend on the context of the question. Children can also answer questions where the quotient needs to be rounded according to the context.

			2	4	r	1	2
1	5	3	7	2			
	–	3	0	0			
			7	2			
	–		6	0			
			1	2			

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

			2	4	$\frac{4}{5}$
1	5	3	7	2	
	–	3	0	0	
			7	2	
	–		6	0	
			1	2	

Glossary

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor

Key Vocabulary

Foundation		
Adding and Subtracting add, more, and make, sum, total altogether score double one more, two more, ten more... how many more to make... ? how many more is... than...? take (away), leave how many are left/left over? how many have gone? one less, two less... ten less... how many fewer is... than...? difference between is the same as	**Solving problems.** **Reasoning about numbers or Shapes** pattern puzzle answer right, wrong what could we try next? how did you work it out? count, sort group, set match same, different list	**Problems involving 'real life' or money** compare double half, halve pair count out, share out left, left over money coin penny, pence, pound price cost buy sell spend, spent pay change dear, costs more cheap, costs less, cheaper costs the same as how much...?

		how many...? total
Year 1		
Addition and subtraction +, add, more, plus make, sum, total altogether score double, near double one more, two more... ten more how many more to make...? how many more is... than...? how much more is...? -, subtract, take (away), minus leave how many are left/left over? how many are gone? one less, two less, ten less... how many fewer is... than...? how much less is...? difference between half, halve =, equals, sign, is the same as	**Multiplication and Division** lots of, groups of x, times, multiply, multiplied by once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of ÷, divide, divided by, divided into, left, left over	**Solving Problems** **Making decisions and reasoning** pattern puzzle answer right, wrong what could we try next? how did you work it out? count out, share out, left, left over number sentence sign, operation

Year 2		
Addition and Subtraction +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make...? how many more is... than...? how much more is...? -, subtract, take away, minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many less is... than...? how much fewer is...? difference between half, halve =, equals, sign, is the same as tens boundary	**Multiplication and Division** lots of, groups of x, times, multiply, multiplied by, multiple of once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of ÷, divide, divided by, divided into, left, left over	**Solving Problems** **Making decisions and reasoning** pattern, puzzle calculate, calculation mental calculation jotting answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol

Year 3		
Addition and Subtraction +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make ...? how many more is... than ...? how much more is...? -, subtract, take (away), minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than ...? how much less is...? difference between half, halve =, equals, sign, is the same as tens boundary, hundreds boundary	**Multiplication and division** lots of, groups of x, times, multiplication, multiply, multiplied by, multiple of, product once, twice, three times, four times, five times... ten times... times as (big, long, wide and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of ÷, divide, division, divided by, divided into left, left over, remainder	**Solving problems** **Making decisions and reasoning** pattern, puzzle calculate, calculation mental calculation method jotting answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation

Year 4		
Addition and subtraction add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...? subtract, subtraction, take away, minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is... than...? how much more/less is...? is the same as, equals, sign tens boundary, hundreds boundary inverse	**Multiplication and division** lots of, groups of times, multiplication, multiply, multiplied by multiple of, product once, twice, three times four times, five times... ten times times as (big, long, wide, and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of divide, division, divided by, divided into, divisible by remainder factor, quotient inverse	**Solving problems** **Making decisions and reasoning** pattern, puzzle calculate, calculation mental calculation method jotting answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation

Year 5		
Addition and subtraction add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...? subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/ fewer is... than...? how much more/less is...? equals, sign, is the same as tens boundary, hundreds boundary units boundary, tenths boundary inverse	**Multiplication and Division** lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times four times, five times... ten times times as (big, long, wide, and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of divide, divided by, divided into, divisible by, divisor remainder factor, quotient, divisible by inverse long division / multiplication short division / multiplication	**Solving Problems** **Making decisions and reasoning** pattern, puzzle calculate, calculation mental calculation method, strategy jotting answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation

Year 6		
Addition and subtraction add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...? subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is... than...? how much more/less is...? is the same as, equals, sign tens boundary, hundreds boundary units boundary, tenths boundary inverse amount brackets calculator: clear, display, enter, key, memory, change (money) commutative complements (in 10, 100) currency discount	**Multiplication and division** lots of, groups of times, multiplication, multiply, multiplied by multiple of, product once, twice, three times four times, five times... ten times times as (big, long, wide, and so on) repeated addition array, row, column double, halve share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of divide, division, divided by, divided into remainder factor, quotient, divisible by inverse divisible by, divisor remainder long division / multiplication short division / multiplication	**Solving problems** **Making decisions and reasoning** pattern, puzzle calculate, calculation mental calculation method, strategy jotting answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation

exact, exactly exchange rate most/least significant digit		